```
> restart;
> f:=(x,y)->piecewise(x=0 and y=0,0,x*(x^2-y^2)/(x^2+y^2));
```

$$f := (x, y) \rightarrow piecewise\left(x = 0 \text{ and } y = 0, 0, \frac{x\left(x^2 - y^2\right)}{x^2 + y^2}\right)$$

```
> Delta:=f(1+h,1+k)-f(1,1);
```

$$\Delta := \frac{(1 + h)\left((1 + h)^2 - (1 + k)^2\right)}{(1 + h)^2 + (1 + k)^2}$$

```
> Diff(f(x,y),x)=diff(f(x,y),x);
```

$$\frac{\partial}{\partial x}\left(\frac{x\left(x^2 - y^2\right)}{x^2 + y^2}\right) = \frac{x^2 - y^2}{x^2 + y^2} + \frac{2x^2}{x^2 + y^2} - \frac{2x^2\left(x^2 - y^2\right)}{\left(x^2 + y^2\right)^2}$$

```
> Diff(f(x,y),y)=diff(f(x,y),y);
```

$$\frac{\partial}{\partial y}\left(\frac{x\left(x^2 - y^2\right)}{x^2 + y^2}\right) = -\frac{2xy}{x^2 + y^2} - \frac{2x\left(x^2 - y^2\right)y}{\left(x^2 + y^2\right)^2}$$

```
> fx11:=limit((f(1+x,1)-f(1,1))/x,x=0);
```

$$fx11 := 1$$

```
> fy11:=limit((f(1,y+1)-f(1,1))/y,y=0);
```

$$fy11 := -1$$

```
> fx:=(x,y)->(x^2-y^2)/(x^2+y^2)+2*x^2/(x^2+y^2)-2*x^2*(x^2-y^2)/
  (x^2+y^2)^2;
```

$$fx := (x, y) \rightarrow \frac{x^2 - y^2}{x^2 + y^2} + \frac{2x^2}{x^2 + y^2} - \frac{2x^2\left(x^2 - y^2\right)}{\left(x^2 + y^2\right)^2}$$

```
> fy:=(x,y)->-2*x*y/(x^2+y^2)-2*x*(x^2-y^2)*y/(x^2+y^2)^2;
```

$$fy := (x, y) \rightarrow -\frac{2xy}{x^2 + y^2} - \frac{2x\left(x^2 - y^2\right)y}{\left(x^2 + y^2\right)^2}$$

```
> fx(1,1);
```

$$1$$

```
> fy(1,1);
```

$$-1$$

```
> (f(1+h,1+k)-f(1,1))-(1*h + (-1)*k);
```

$$\frac{(1+h)\left((1+h)^2-(1+k)^2\right)}{(1+h)^2+(1+k)^2}-h+k$$

```
> simplify(%);
```

$$\frac{h^2+k^2-2\,h\,k-2\,h\,k^2+k\,h^2+k^3}{2+2\,h+h^2+2\,k+k^2}$$

```
> eta[1]:=(h,k)->1/2*(h^2+k^2-2*h*k-2*h*k^2+k*h^2+k^3)/(h*(2+2*h+
  h^2+2*k+k^2));
```

$$\eta_1:=(h,k)\to\frac{1}{2}\,\frac{h^2+k^2-2\,h\,k-2\,h\,k^2+k\,h^2+k^3}{h\left(2+2\,h+h^2+2\,k+k^2\right)}$$

```
> eta[2]:=(h,k)->1/2*(h^2+k^2-2*h*k-2*h*k^2+k*h^2+k^3)/(k*(2+2*h+
  h^2+2*k+k^2));
```

$$\eta_2:=(h,k)\to\frac{1}{2}\,\frac{h^2+k^2-2\,h\,k-2\,h\,k^2+k\,h^2+k^3}{k\left(2+2\,h+h^2+2\,k+k^2\right)}$$

```
> h*eta[1](h,k)+k*eta[2](h,k);
```

$$\frac{h^2+k^2-2\,h\,k-2\,h\,k^2+k\,h^2+k^3}{2+2\,h+h^2+2\,k+k^2}$$

```
> Limit(eta[1](h,k),{h=0,k=0})=limit(eta[1](h,k),{h=0,k=0});
```

$$Limit\left(\frac{1}{2}\,\frac{h^2+k^2-2\,h\,k-2\,h\,k^2+k\,h^2+k^3}{h\left(2+2\,h+h^2+2\,k+k^2\right)},\{h=0,k=0\}\right)$$

$$=limit\left(\frac{1}{2}\,\frac{h^2+k^2-2\,h\,k-2\,h\,k^2+k\,h^2+k^3}{h\left(2+2\,h+h^2+2\,k+k^2\right)},\{h=0,k=0\}\right)$$

```
> Limit(eta[1](r*cos(theta),r*sin(theta)),r=0)=limit(eta[1](r*cos
  (theta),r*sin(theta)),r=0);
```

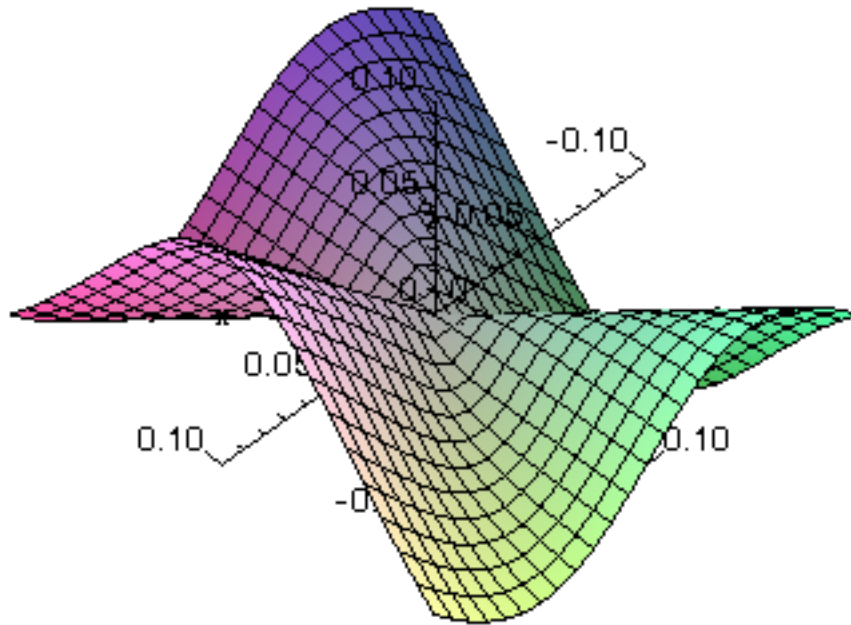$$\lim_{r\to 0}\frac{1}{2}\,\left(r^2\cos(\theta)^2+r^2\sin(\theta)^2-2\,r^2\cos(\theta)\sin(\theta)-2\,r^3\cos(\theta)\sin(\theta)^2\right.$$

$$+r^3\sin(\theta)\cos(\theta)^2+r^3\sin(\theta)^3\Big)\Big/\left(r\cos(\theta)\left(2+2\,r\cos(\theta)+r^2\cos(\theta)^2+2\,r\sin(\theta)\right.\right.$$

$$\left.\left.+r^2\sin(\theta)^2\right)\right)=0$$

```
> Limit(eta[2](r*cos(theta),r*sin(theta)),r=0)=limit(eta[2](r*cos
  (theta),r*sin(theta)),r=0);
```

$$\lim_{r\to 0}\frac{1}{2}\,\left(r^2\cos(\theta)^2+r^2\sin(\theta)^2-2\,r^2\cos(\theta)\sin(\theta)-2\,r^3\cos(\theta)\sin(\theta)^2\right.$$

$$+r^3\sin(\theta)\cos(\theta)^2+r^3\sin(\theta)^3\Big)\Big/\left(r\sin(\theta)\left(2+2\,r\cos(\theta)+r^2\cos(\theta)^2+2\,r\sin(\theta)\right.\right.$$

$$\left.\left.+r^2\sin(\theta)^2\right)\right)=0$$

```
> with(plots):
> plot3d(f(x,y),x=-0.1..0.1,y=-0.1..0.1,axes=normal);
```

```
> f:=(x,y)->x^2+2*y*x^2;
```
$$f := (x, y) \rightarrow x^2 + 2\,y\,x^2$$

```
> g:=t->t^2-3*t+1;
```
$$g := t \rightarrow t^2 - 3\,t + 1$$

```
> h:=t->exp(-t)+t;
```
$$h := t \rightarrow e^{-t} + t$$

```
> f(g(t),h(t));
```
$$\left(t^2 - 3\,t + 1\right)^2 + 2\left(e^{-t} + t\right)\left(t^2 - 3\,t + 1\right)^2$$

```
> diff(%,t);
```
$$2\left(t^2 - 3\,t + 1\right)(2\,t - 3) + 2\left(-e^{-t} + 1\right)\left(t^2 - 3\,t + 1\right)^2 + 4\left(e^{-t} + t\right)\left(t^2 - 3\,t + 1\right)(2\,t - 3)$$

```
> simplify(%);
```
$$-44\,t^3 + 48\,t^2 - 2\,t - 4 - 2\,e^{-t}\,t^4 + 20\,e^{-t}\,t^3 - 58\,e^{-t}\,t^2 + 56\,e^{-t}\,t - 14\,e^{-t} + 10\,t^4$$

```
> diff(f(x,y),x)*diff(g(t),t)+diff(f(x,y),y)*diff(h(t),t);
```

$$(2x + 4yx)(2t - 3) + 2x^2(-e^{-t} + 1)$$

```
> subs(x=t^2-3*t+1,y=exp(-t)+t,%);
```
$$\left(2t^2 - 6t + 2 + 4\left(e^{-t} + t\right)\left(t^2 - 3t + 1\right)\right)(2t - 3) + 2\left(-e^{-t} + 1\right)\left(t^2 - 3t + 1\right)^2$$

```
> simplify(%);
```
$$-44t^3 + 48t^2 - 2t - 4 - 2e^{-t}t^4 + 20e^{-t}t^3 - 58e^{-t}t^2 + 56e^{-t}t - 14e^{-t} + 10t^4$$

```
> g:=x^2-3*x*y^3-1=0;
```
$$g := x^2 - 3xy^3 - 1 = 0$$

```
> implicitdiff(g,x,y);
```
$$-\frac{9xy^2}{-2x + 3y^3}$$

```
> implicitdiff(g,y,x);
```
$$\frac{1}{9}\frac{2x - 3y^3}{xy^2}$$

```
> g:=(x,y)->x^2-3*x*y^3-1;
```
$$g := (x, y) \rightarrow x^2 - 3xy^3 - 1$$

```
> yprime:=-diff(g(x,y),x)/diff(g(x,y),y);
```
$$yprime := \frac{1}{9}\frac{2x - 3y^3}{xy^2}$$

```
> f := y^2-2*x*z = 1;
```
$$f := y^2 - 2xz = 1$$

```
> g := x^2-exp(x*z) = y;
```
$$g := x^2 - e^{xz} = y$$

```
> implicitdiff({f,g},{y,z},y,x);
```
$$\frac{2x}{ye^{xz} + 1}$$

```
> implicitdiff({x^2+y=z, x+y*z=1},{y,z},{y,z}, x) ;
```
$$\left\{D(y) = -\frac{2yx + 1}{z + y}, D(z) = \frac{-1 + 2xz}{z + y}\right\}$$

```
> implicitdiff({x^2+y=z, x+y*z=1}, {y,z}, y, x);
```
$$-\frac{2yx + 1}{z + y}$$

```
> f := x^3*y-2*y/z=z^2;
```
$$f := x^3 y - \frac{2y}{z} = z^2$$

```
> implicitdiff(f,y(x,z),x);
```

$$-\frac{3\,x^2\,y\,z}{x^3\,z-2}$$

```
> implicitdiff(f,y(x,z),x,z);
```

$$-\frac{6\,x^2\left(-2\,y+z^3\right)}{x^6\,z^2-4\,x^3\,z+4}$$

```
> f := x^2+y^2+z^2=0;
```

$$f := x^2+y^2+z^2=0$$

```
> implicitdiff(f,y(x,z),x);
```

$$-\frac{x}{y}$$

```
> implicitdiff(f,y(x,z),x,z);
```

$$-\frac{x\,z}{y^3}$$